Insurance Versus Self-Insurance: A Risk Management Perspective

Patrick L. Brockett, Samuel H. Cox, Jr. and Robert C. Witt

ABSTRACT

The scientific risk-retention or self-insurance decision from a utility theoretic point of view is examined under the assumption that the risk manager has only partial stochastic information about the loss severity distribution. When only the range and a few central moments are known about the loss severity distribution, it is shown how to obtain maximally tight bounds on the expected utility of self insurance. Significantly, the extremal probability distributions derived do not depend upon the particular decision maker's utility function and, therefore, should be applicable to a wide variety of financial decisions. Moreover, financial and/or risk managers in a business can make decisions without assessing the preference structure of the firm's owners.

The financial decision as to whether a business firm should self-insure a group of exposure units is a complicated but very important one. Many exposures to loss are so financially inconsequential that they can be safely retained and paid out of normal cash flow. However, other exposures to loss that have large financial consequences cannot be retained by the firm without a

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Patrick L. Brockett is Associate Professor of Actuarial Science, Richard Seaver Contenial Fellow, and Paul V. Montgomery Centennial Fellow in Actuarial Science in the Department of Finance and the Applied Research Laboratories, University of Texas at Austin. He earned a Ph.D. in Mathematics at the University of California — Irvine.

Samuel H. Cox, Jr. is also an Associate Professor and Paul V. Montgomery Centennial Fellow in Actuarial Science at the University of Texas. He earned a Ph.D. in Mathematics at Louisiana State University and is a Fellow of the Society of Actuaries.

Robert Witt is Chairman of the Finance Department and the Joseph H. Blades Centennial Professor of Risk Management and Insurance at the University of Texas. He is a member of the *Journal*'s Editorial Board and currently President of the American Risk and Insurance Association.

All three authors have published articles previously in this *Journal*. Professor Witt received the 1984 Alpha Kappa Psi Award for the most outstanding paper published 10 years earlier in *The Journal of Risk and Insurance*.

The authors greatly appreciate some thoughtful suggestions provided by Emilio Venezian.

detailed scientific analysis of the financial implications of such a decision.[1] Due to the complexity and financial importance of this decision, risk managers have started to utilize some advanced financial and statistical tools to assess the risks involved with the retention of various loss exposures, such as workers' compensation, products liability, and other liability exposures.[2]

Risk managers have started to rely on the loss retention method more frequently because of cash flow considerations and the high interest rates available in recent years. Loss exposures that were frequently insured commercially before are now retained by the business firm. If the number of exposure units is sufficient to estimate the expected losses and associated financial risks and a reserve fund is established for the expected and unexpected retained losses, this arrangement is basically a form of self-insurance.[3] By eliminating some of the transactions costs associated with commercial insurance (such as commissions and premium taxes), firms can reduce their expected loss and expense costs, which would be consistent with a firm goal of maximizing expected profits and shareholder wealth.

Risk management is a specialized area of financial management where one is concerned with scientific financing of contingent future claims (losses) which may have an adverse impact on cash flows, profit, and the value of the firm. The pure loss scientific financing mechanisms considered in this paper are self-insurance and insurance. Both of these mechanisms are pre-loss types of financing arrangements. Post-loss financing mechanisms would involve the use of lines of credit or contingent loans that would be dependent upon the occurrence of a loss to the firm in the future.[4]

Risk management decisions are quite similar to other financial decisions, but the primary focus of risk management is on potential losses or pure risks, rather than on potential gains or speculative risks, and on how to finance

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[1] The Johns-Manville bankruptcy filing is a topical case in point. This corporation filed for protection under the bankruptcy laws due to potential liability losses associated with asbestosis which were greater than their net worth of $1.2 billion at the time of filing.

[2] Rapidly increasing insurance rates during the past decade have encouraged risk managers to consider alternatives to insurance, such as larger deductibles and self-insurance. Recently, the United States Congress helped to stimulate this risk management evolution by passing the Products Liability Risk Retention Act in 1981. This law allows companies to self-insure their product liability exposures. Thus, the risk management decision model developed here should be useful to many business firms.

[3] It is interesting to note that a recent study by Dickson [7] of the ability of corporate financial managers to predict profit and loss distributions found the following. Traditionally-trained financial managers and risk-management-trained or experienced managers did equally well in predicting profit distributions and positive cash flows. However, when it came to assessing possible loss distributions, those managers trained or experienced in risk management were significantly more risk averse than their traditionally trained financial counterparts. This difference in perceptions would seem to have important implicatons for the perceived value of projects. Accordingly, the techniques presented here should be of use to financial managers for assessing loss distributions in a quantitative manner.

[4] See Doherty [8] for an interesting discussion of contingency loans to finance losses.

potential liability, property, and personnel losses of the firm.[5] The analysis here is restricted to pure risks and financial aspects of risk management.

Risk management decisions should be made on a basis broadly consistent with other financial decisions so as to maximize a risk-averse owner's expected utility or his wealth. It is assumed that an owner or owners of a closely held business firm communicate a risk-return tradeoff policy to their risk manager which is consistent with a risk-averse utility function.[6] In the simple situation considered here, the risk manager (possibly the owner) in many cases can choose between two "projects": insurance and self-insurance. The decision method is based on the expected-utility paradigm and gives bounds (which can be calculated at the time the decision is made in terms of moments of the loss distribution) on the expected utility of self-insurance. Another simplifying assumption used throughout this paper is that all losses are to be paid at the end of the period and premiums are paid at the beginning of the period. In such a single-period setting, the losses can be discounted at the firm's hurdle rate of return or cost of capital to the beginning of the year. This allows suppressing further reference to the firm's cost of capital by assuming premiums and losses are evaluated at the same point in time. Thus, losses in the following models may be viewed as the appropriate discounted values, which simplifies the presentation and allows one to focus on the pure risk-management aspects of the problem.

Since insurance is a pre-loss financing arrangement, an opportunity cost is incurred when the premium is paid for this type of financing for contingent losses. The costs of covering uncertain future losses, however, are transferred to the insurer, who is a financial intermediary specializing in diversifying pure losses among a group of similarly situated business firms. In this regard, insurance basically involves a pure risk transfer for a price that can be viewed as a pre-loss financing method for future contingent claims.

Self-insurance can also be viewed as a scientific pre-loss financing arrangement if funds are set aside in a relatively liquid form to cover expected and unexpected losses. The opportunity cost involved here is the difference between the return on alternative available investments in the firm and the lower return on the liquid self-insurance fund. Of course, the funds set aside

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[5] Recent studies of the psychology of preferences have found some interesting discrepancies between how individuals perceive pure and speculative risks and how these differences affect their decision processes. See Kahneman and Tversky [13].

[6] The analysis is not intended to apply to large publicly-held corporations with diffuse ownership. Modern finance theory suggests stockholders in such corporations can hedge or eliminate pure or insurable risks through diversification of their portfolios of financial assets. Mayers and Smith [17] indicate that the purchase of insurance by such corporations would reduce stockholder wealth because it would represent a negative net present value project. The demand for insurance by large widely-held corporations cannot be satisfactorily explained by risk aversion, according to Mayers and Smith [17]. They suggest a set of incentives for the corporate purchase of insurance which does not include risk aversion, except for closely-held corporations.

may not cover potential losses. Losses could exceed the self-insurance fund, which means that the firm would have to raise additional funds at the time of a major loss. Special financing arrangements might be necessary if actual losses exceeded funds reserved for expected and unexpected losses during some planning period. If this difference were relatively large, such losses could have an adverse impact on the market price of a publicly-traded firm's stock (see Sprecher and Pertl [19]). Nevertheless, self-insurance can be viewed as a form of internal diversification within a business firm when the number of exposure units is sufficiently large for the risk manager to predict losses within some reasonable bounds.

Insurance, risk management, and financial literature usually indicates that several requisites should be met before a firm decides to self-insure some type of exposure to loss. These requirements include the following: First, the firm should have a large number of exposure units that are not subject to simultaneous destruction or a catastrophic hazard. Second, the firm should be in good financial condition such that it is able to meet large and unusual losses either from working capital or from reserve funds established to cover anticipated and unanticipated losses. If a firm cannot afford to pay for commercial insurance premiums, it is highly unlikely that it could pay for large losses if they should occur (the bankruptcy risk should not be ignored).

In this paper, the assumed risk-return tradeoff policy followed by the firm's risk manager will be based on an expected utility approach. "Worst" and "best" possible case distributions are explicitly derived, and tight upper and lower bounds on the expected gain or loss of utility resulting from self-insurance are obtained. The technique can be used to obtain information concerning the number of exposure units the company must have for self-insurance to be financial feasible and, with some modifications of the formulation derived below could be potentially used for assessing the chance of bankruptcy.

The expected utility model is then used to translate utility bounds into dollar bounds on the amount which a risk-averse business firm should be willing to pay for insuring a potential loss when the firm only has partial information about the loss distribution. These bounds could be useful in negotiating business insurance contracts, as well as for helping a firm to decide when to self-insure. The expected utility bounds could also be used by an owner or a risk manager for a risk-averse firm which wants to maximize expected utility subject to a chance constraint based upon some specific probability of regret.

The Statistical Framework

The statistical framework for analyzing the expected utility of self-insurance is developed in this section.

A general non-specific exposure unit could be utilized, but for concreteness an automobile exposed to the collision or liability peril during one year will frequently be cited to as a specific exposure unit. Clearly, disability insurance, products liability, or other types of insurance could also be used. Let X_i

be defined as a random variable which denotes the discounted value of the total loss for the ith exposure unit during some given policy or exposure period, such as one year. The discounted total loss for an exposure unit is a function of the number of accidents and the size or severity of loss associated with each accident, that is:

$$X_i = \sum_{j=0}^{N_i} Y_{i,j} \tag{1}$$

where $Y_{i,j}$ is the random amount of discounted loss for the jth accident of the ith exposure unit, and N_i is the random number of accidents incurred by the ith exposure unit.

The random variables $Y_{i,j}$ are assumed to be independently and identically distributed for the ith exposure unit or automobile. This means that each automobile is assumed to have a given severity distribution, but the losses from different accidents are independent. However, this does not imply that every exposure unit i must have the same severity distribution. For example, model differences in automobiles could be recognized with different severity distributions in order to allow for differences in collision losses among these exposure units. It is also assumed that the severity distribution for a given exposure unit is independent of that of all other exposure units and that the number of accidents for the ith exposure, N_i, is independent of all other exposure units. Therefore, the total loss distributions for the exposure units are independent, but they need not be assumed to be identically distributed.

Since homogeneous exposure units have not been assumed, the exposure units in the group under consideration can have different expected losses and premiums. For example, the fleet of automobiles being considered for self-insurance may include many different models. More generally, this means that the total loss distribution may correspond to entirely different exposure units associated with a business; for example, it could include products liability, collision, workers' compensation, and medical expense exposures. Thus, it is not necessary to assume completely homogeneous exposure units for the purpose of this analysis.

The premium or gross rate for the ith exposure unit will be denoted by R_i. The total premium paid for a group of n exposure units will be denoted as R, which is merely the sum of the premiums for the individual exposure units in the group:

$$R = \sum_{i=1}^{n} R_i . \tag{2}$$

A finite upper bound, b_i, on the severity of any loss will be assumed.[7] If automobile liability insurance were being considered, the limits of liability on a commercial policy would constrain the amount of loss per accident to b_i, so for comparisons with self-insurance it must be considered. Of course, one exposure unit could have more than one accident, so the upper bound on the total loss distribution for the ith exposure unit could be greater than the upper bound, b_i, on the severity distribution.

The Expected Gain or Loss in Utility from Self Insurance

The decision to self-insure should recognize not only the chance of regret (or satisfaction) if losses exceed (do not exceed) the premium that could have been paid for insurance[8] (or more generally, the chance of losses exceeding or being less than a given amount, such as a given percentage of the equity of the firm), but also the actual monetary gain or loss and the associated utility or disutility. The probability that losses exceed or are less than a given amount can be assessed using the techniques outlined in Brockett, Cox and Witt [5]. However, a more general measure of risk or disutility is still needed and is developed below.

This section shows how to obtain sharp upper and lower bounds on the expected gain or loss in utility associated with self-insurance. Moreover, it demonstrates that the expected utility bounds can be translated into acceptable and unacceptable dollar amounts for insuring a risky venture using only the available limited information concerning the stochastic nature of the loss variables.

Let $U(x)$ denote the utility function or risk-return policy of the potential insured firm and W_0 its wealth at the beginning of its planning period.[9] If each of n exposure units produces a loss of X_i, as defined in (1), the total losses for a firm with n exposures can be denoted by (3).

$$X = \sum_{i=1}^{n} X_i \tag{3}$$

[7] Mathematically, the precise value of b_i, has some effect on the results derived from the expected utility calculation, as would be expected. However, it should be noted that if b is very large, say roughly equivalent to the initial wealth W_0, then the entire expected utility paradigm breaks down. As Friedman and Savage [11] have shown, when the total wealth level is small, individuals tend to be risk takers rather than risk averters. Also, for a given set of data, the value of b is fixed and determinable, and this is the value of b we recommend for use with this model.

[8] See McWhorter [18] for an analysis of this case.

[9] In a small entrepreneurial business, this utility function of the firm may be the utility function of the owner-manager. This type of firm encompasses over 95 percent of the business firms in the United States, which generate over 70 percent of the sales revenue, see Walker [20]. For larger, closely-held corporations, the directors may pass their desires to an agent-manager. If these directives are interpreted by the agent-manager to be consistent with a risk-averse utility function, this utility function may be used in the calculations. A risk-neutral utility function would just be a special case in the model, which might be appropriate for a large widely-held corporation.

As previously defined in (2), total premiums of R for the policy period are:

$$R = \sum_{i=1}^{n} R_i$$

where R_i is the premium for insuring the ith exposure unit.

The indifference premium or certainty equivalent is defined to be that amount, R*, which the corporation would be indifferent between paying to the insurer to assume the risk of loss X or retaining the loss exposure itself. The indifference premium is found by solving the equation in (4) for R*,

$$U(W_0 - R^*) = E[U(W_0 - X)] \qquad (4)$$

where W_0 is the initial wealth level. A decision rule can now be specified. If the quoted premium R is below R*, the business firm should buy the insurance, while, if R is greater than R*, it should choose to retain or self-insure the loss exposure.[10] Of course, knowing the value of R* would also be useful for negotiating a new insurance contract.

Assume that the only knowledge available to the potential insured concerns the first three moments μ, σ^2 and ρ for the severity distribution (this information could be obtained, for example, by using internal company data, or industry-wide averages). Other than this assumed information, no assumption about the form of the severity distribution is made. It is of unknown distributional form. Now, the problem is to find upper and lower bounds on the indifference premium R* using only the above information. Since R* is a function of $E[U(W_0 - X)]$, that is:

$$R^* = W_0 - U^{-1}(E[U(W_0 - X)]) \qquad (5)$$

attention can be focused on finding bounds for:

$$E[U(W_0 - X)] = E[U(W_0 - \sum_{i=1}^{n} \sum_{j=0}^{N_i} Y_{ij})].$$

To formulate the problem mathematically, let F denote the collection of all probability distributions for loss severity on the interval [a,b] which have a mean μ, variance σ^2, and third central moment ρ (a skewness measure).

Based on a remarkable result of the Markov-Krein Theorem concerning Tchebychev systems of functions it can be shown that E(h(Y)) has a maximum and minimum as the distribution of Y ranges over F as long as the fourth derivative of h is greater than zero, $h^{(4)}(x) > 0$. Even more remarkable is the fact that the extreme measures in F do not depend in any way upon h. The following theorem is stated explicitly in Brockett [2] and follows im-

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[10] If the firm were risk neutral and the insurer and firm had perfect symmetrical information, R would always be greater than R* and the firm would never buy insurance.

mediately from the general Markov-Krein theorem presented in Karlin and Studden [14].[11]

Theorem 1 If $h^{(4)}(x) > 0$, then

i) max $E[h(X)]$ is obtained by the distribution in **F** with probability p_1 at a, p_2 at ξ, and $(1 - p_1 - p_2)$ at b where

$$\xi = \frac{\rho - (a+b-2\mu)\sigma^2}{(a-\mu)(b-\mu) + \sigma^2} + \mu$$

$$p_1 = \frac{\sigma^2 + (\xi-\mu)(b-\mu)}{(b-a)(\xi-a)} \tag{6}$$

and

$$p_2 = \frac{\sigma^2 + (b-\mu)(a-\mu)}{(\xi-b)(\xi-a)}$$

ii) min $E[h(X)]$ is obtained by the distribution in **F** with probability q at η_1 and $(1 - q)$ at η_2 where

$$q = \frac{1}{2} + \frac{\rho.}{2\sqrt{\rho^2 + 4\sigma^6}}$$

$$\eta_1 = \mu + \frac{\rho - \sqrt{\rho^2 + 4\sigma^6}}{2\sigma^2} \tag{7}$$

and

$$\eta_2 = \mu + \frac{\rho + \sqrt{\rho^2 + 4\sigma^6}}{2\sigma^2}$$

Intuitively, when h represents a utility function, the previous theorem delineates a "best possible case" and a "worst possible case" for the distribution of the random variable under consideration. This information is important for managerial modeling. Once the support points of the extreme probability measures are known, the three-moment constraint equations can be solved simultaneously to find the probabilities given in (6) and (7). The distribution in (6) is called the *upper principal representation*, and the one in (7) is defined as the *lower principal representation* for **F**.

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[11] This theorem is of independent interest in economic and financial modeling, as is shown in Brockett [2].

Based on this theorem, the desired bounds on the expected gain or loss in utility can now be found. For convenience and simplicity of presentation, it shall be assumed that both the N_i's and the Y_{ij}'s are identically distributed for each exposure unit. Define $S_0 = 0$ for $k = 0$, and for $k \geq 1$, let S_k be a random variable whose distribution is that of the sum of k independent Y's. Let N be defined as the following sum:

$$N = \sum_{i=1}^{n} N_i \tag{8}$$

which represents the total number of losses for the group of n exposure units. The probabilities $P[N = k]$ can be calculated by using the probability generating function below:

$$\Phi(\theta) = E(\theta^N) = \sum_{j\geq 0} p_j \theta^j$$

to generate specific probability values in the following way

$$P[N=k] = \left.\frac{d^k \Phi(\theta)}{d\theta^k \, k!}\right|_{\theta=0}$$

To calculate $E[U(W_0 - X)]$, it is first necessary to condition on N to obtain:

$$E(E[U(W_0 - \sum_{i=1}^{n} \sum_{j=0}^{N_i} Y_{ij})|N]) = \sum_{k\geq 0} P[N=k]E[U(W_0-S_k)].$$

Now Theorem 1 can be applied to obtain bounds on $E[U(W_0 - S_k)]$. If $h(x) = -U(W_0-x)$ is four times differentiable, then Theorem 1 can be applied. Note that for most utility functions postulated in the literature (e.g., exponential utility, logarithmic utility, power utility, etc.) they actually have $U(x) > 0$. $U'(x) > 0$, $U''(x) < 0$, $U^{(3)}(x) > 0$, $U^{(4)}(x) < 0$. Thus, by Theorem 1,

$$E[h(S_k)] = -E[U(W_0-S_k)]$$

can be bounded above and below by the moments of S_k. Since these moments for independent variables are additive, the moments for S_k are $k\mu$, $k\sigma^2$, and $k\rho$, respectively. Remembering that a minus sign relates h and U, which will reverse the extremal distributions in Theorem 1, the following relationship is obtained:

$$\begin{aligned} &U(W_0)p_1^{(k)} + U(W_0-\xi^{(k)})p_2^{(k)} + U(W_0-kb)(1-p_1^{(k)}-p_2^{(k)}) \\ &\leq E[U(W_0-S_k)] \leq U(\eta_1^{(k)})q^{(k)} + U(\eta_2^{(k)})(1-q^{(k)}) \end{aligned} \tag{9}$$

where $\rho_1^{(k)}$, $\xi^{(k)}$, $\rho_2^{(k)}$ are given by (6), and $\eta_1^{(k)}$, $\eta_2^{(k)}$, and $q^{(k)}$ are given by (7) with $k\mu$, $k\sigma^2$, $k\rho$, kb and 0 replacing μ, σ^2, ρ, b and a, respectively.

Multiplying equation (9) through by $P[N = k]$ and summing gives upper and lower bounds on $E[U(W_0 - X)]$. Although the above formulas may appear complicated at first glance, they are easily programmed, and since the probabilities P[N = k] decrease rather rapidly, only a relatively few terms must be carried in the summation to obtain a given decimal accuracy. One may then easily calculate the indifference premium specified in (5) for the upper and lower bounds. The lower bound on the indifference premium is a level below which the corporation will always insure, and the upper bound is the amount above which the corporation will always self-insure. Between the upper and lower bounds, information on the loss structure is insufficient to make an unequivocal decision.

Alternately, it is often the case that the distribution of N and the utility function allow for explicit summation and the development of concrete algebraic formulas for the bounds. Moreover, since the upper and lower principal representations do not depend upon the utility function, the exact values of the utility function need only be assessed at the two extremes, a and b, and at one other previously determined point. This important fact simplifies actual implementation of utility-based decision rules. Traditional approaches to expected utility require that the exact distribution of X must be known and that the utility function must be assessed at all wealth levels, which has made the theory basically inoperable in practice.

For the risk manager, the fact that the "best possible" and "worst possible" case probability distributions do not depend upon the utility function of the owner or equity holders means that decisions can be made without directly assessing the preference structure of the owner or owners. This fact greatly simplifies the decision-making process for the risk manager.

To illustrate the previous results in a simplified manner, numerical results shall be presented using some real data on claims frequency N_i and moments of the severity distribution.

Assume the N_i's have a Poisson distribution with the same parameter (expected frequency) λ for each exposure unit, that is,

$$P[N_i=k] = e^{-\lambda}\lambda^k/k!$$

For this example, assume that the insured's utility function is exponential:

$$U(x) = -e^{-\gamma x}.$$

The parameter $\gamma = -U''(x)/U'(x)$ is the Arrow-Pratt measure of absolute risk aversion, which is constant and does not depend upon wealth. The assumption that the insured has an exponential utility can be supported by various arguments, which have been developed in Cozzolino [6], Freifelder [9], and Brockett [3].[12]

From the Poisson assumption, it follows that the total number of losses

$$N = \sum_{i=1}^{n} N_i$$

[12] Brockett [3] has shown that any of the commonly used utility functions can be obtained as simple mixtures of exponential utility functions.

is Poisson with a mean value of $n\lambda$. Moreover, the probability of exactly k losses is

$$P[N=k] = \frac{e^{-n\lambda}(n\lambda)^k}{k!} .$$

The corresponding term $E[U(W_0 - S_k)]$ is specified below:

$$E[-e^{-\gamma(W_0-S_k)}] = -e^{-\gamma W_0}E[e^{\gamma S_k}] = -e^{-\gamma W_0}[E(e^{\gamma Y})]^k .$$

Thus, the following result is obtained:

$$E[U(W_0 - \sum_{i=1}^{n}\sum_{j=0}^{N_i} Y_{ij})] = -\sum_{k=0}^{\infty} P[N=k]e^{-\gamma W_0}(E(e^{\gamma Y}))^k$$

$$= -e^{-\gamma W_0}\sum_{k=0}^{\infty}\frac{e^{-n\lambda}(n\lambda E(e^{\gamma Y}))^k}{k!}$$

$$= -\exp\{-\gamma W_0 - n\lambda + n\lambda E(e^{\gamma Y})\} = g(E(e^{\gamma Y}))$$

where $g(x) = -\exp\{-\gamma W_0 - n\lambda + n\lambda x\}$ is a monotonically decreasing function. Thus, the best possible bounds for the expected utility are found by finding the best possible bounds for $E(e^{\gamma Y})$ for $Y \in F$ and then plugging these bounds into the function g. The best bounds for $E(e^{\gamma Y})$ follow from Theorem 1 by using $h(x) = e^{\gamma x}$, and $a = 0$. In fact, the upper bound has already been calculated in (3). Summarizing, one obtains the following relations:

$$g(A) \leq E[U(W_0 - \sum_{i=1}^{n}\sum_{j=0}^{N_i} Y_{ij})] \leq g(B) \qquad (10)$$

where $g(x) = -\{\exp - \gamma W_0 - n\lambda + n\lambda x\}$,

$$A = p_1 + e^{\gamma\xi}p_2 + e^{\gamma b}(1-p_1-p_2)$$

$$B = e^{\gamma\eta_1}q + e^{\gamma\eta_2}(1-q), \text{ and}$$

with ξ, p_1 and p_2 given by (6) and η_1, η_2, and q given by (7). Moreover, these bounds cannot be improved; i.e., they are the best attainable bounds with the given information. The only possible way to improve these bounds is to obtain more information about the severity distribution (e.g., perhaps by conducting some extensive statistical work). At any rate, the risk manager can observe the obtainable bounds and make this choice based upon the perceived value of the information.

Having obtained the explicit bounds in (10) for the expected utility of retaining the loss, the corporate risk manager may now obtain an explicit bound on an acceptable premium quote for insuring the loss. By solving the following equation for R*,

$$U(W_0 - R^*) = E[U(W_0 - X)] = g(E(e^{\gamma Y}))$$

and using $U(x) = -\exp(-\gamma x)$, after simplification the following result is obtained.

$$R^* = W_0 - U^{-1}(E[U(W_0 - X)]) = \frac{n\lambda}{\gamma} E[e^{\gamma Y}] - \frac{n\lambda}{\gamma} = \frac{n\lambda}{\gamma}\{E[e^{\gamma Y}] - 1\} \quad (11)$$

The bounds on $E(e^{\gamma Y})$ translate into bounds for the premium. The decision whether to self-insure should now be clear. If the quoted premium R is less than the lower bounds for the indifference premium, R*, *insuring* the loss at R will be expected to increase the owner's utility of wealth, even though the exact form of the loss distribution is not known. On the other hand, if R exceeds the upper bound for R*, then *self-insuring* would be expected to increase the owner's utility, regardless of the true loss distribution. If R is between the upper and lower bounds on R*, some loss distributions will be consistent with the given moments for which self-insurance is optimal and others with these same moments will optimally favor buying insurance at the quoted price R. Since no other information on the loss distribution is available, either the firm's decision to self-insure must be made on other grounds, or additional information must be purchased in order to reduce the size of the insufficient-information interval.

For the numerical illustration presented below, a decision involving whether to self-insure a fleet of automobiles is examined in an expected utility framework. Moments of the severity distribution estimated from the data of the 1963 National Bureau of Casualty Underwriter Study of Automobile Property Damage Liability Insurance Losses by size of claim which were summarized in Witt [21] are used. The first three moments were $\mu = \$139.91$, $\sigma^2 = 38{,}975$, $\rho = 53{,}430{,}000$, and the upper bound on one claim was b = \$5,000. Using the 1958 California Driver Record Study as summarized by Harwayne [12], a Poisson distribution with $\lambda = 0.16$ was fitted to the number of claims per accident.

For illustrative purposes, the Arrow-Pratt measure of risk aversion was specified as $\gamma = 0.001$.[13] The bounds on the indifference premium in equation

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[13] There is some support for the choice of this value for insurance coverage. Using data on insurance coverage of federal employees, Friedman [10] estimated the parameter γ for medical insurance buyers as 0.0025. Keeler, Newhouse and Phelps [15] argued for a lower value and suggested $\gamma = 0.0005$. Our choice of $\gamma = 0.001$ is merely an illustrative compromise between these two values.

(11) are linear functions of the number of exposures, n, or the fleet size here. It was found that R* was in the following interval:

$$\$27.90n \leq R^* \leq \$62.33n \quad (12)$$

when the first two moments were used and that it fell in the interval:

$$\$30.35n \leq R^* \leq \$37.68n \quad (13)$$

when the first three moments were utilized.[14] In either case, for a given fleet size, if the premium offered by the insurer is below the lower bound, the firm should buy the insurance. If the premium quoted is above the upper bound, the firm should reject the bid and self-insure the n exposure units. For premium quotes between the upper and lower bounds, not enough information about the loss distribution exists to make a choice that is always optimal. However, it can be seen that zone of indecision decreases dramatically in size as the number of moments used in the calculation increases from two to three. In the first case, the upper bound is more than double the lower bound; whereas, in the latter case the upper bound is only 24 percent greater than the lower value.[15] These bounds cannot be improved without spending time or money to obtain additional information about the loss distribution for the exposure units being evaluated.[16]

Summary and Conclusions

Both insurance and self-insurance are pre-loss financing arrangements. For insurance, an opportunity cost is incurred when the premium is paid in order to transfer the cost of uncertain future losses to the insurer who specializes in diversifying pure losses among a group of similarly situated business firms for a fixed price. For self-insurance, funds must be set aside in a relatively liquid form to cover future expected and unexpected losses. However, there is no

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[14] The calculation involving two known moments is done using the Markov-Krein Theorem in Karlin and Studden [14] to determine the upper and lower principal representations. The three-moment case used Theorem 1 to obtain tight bounds. The two-moment case formulas can be found in Brockett [2] or Brockett and Cox [4].

[15] Because of the formulas involved in calculating the upper bound, it should be apparent that the value of b does make a difference in the calculation of the upper bound. This is entirely reasonable and expected because as b increases the potential of catastrophic losses becomes more of a consideration. Since our upper bound is a "worst case scenario," the larger the loss limits, the worse the potential loss can be. As an example, in the context of the numerical illustration is this article, if the parameters μ, σ^2 and ρ remain constant, but the loss limit b increases from \$5,000 to \$7,000, the upper value of R* increases to 56,726. For b = 7500, the upper bound on R* increases to 66,842, while b = \$10,000 the upper bound on R* is 228,025. This serves to illustrate that the appropriate selection of b is important for deriving meaningful bounds. This sensitivity analysis varied only b. In practice, if new data were based on a larger limit b, then the parameters μ, σ^2 and ρ might also change.

[16] Information such as unimodality of the loss variable can be incorporated to improve these bounds as well. See Brockett [2], Brockett and Cox [4], or Karlin and Studden [14] for discussion of how to implement this technique in the unimodal case.

guarantee that the funds set aside will cover actual losses in the future. If the losses exceed the self-insurance fund, the firm would have to raise additional funds at the time of a major loss, which might make the firm's owner regret his (her) decision to self-insure. In essence, self-insurance is a form of internal diversification within a business firm which is economically feasible only if the number of exposure units is sufficiently large that losses can be scientifically predicted within some reasonable statistical bounds.

The risk management decision either to insure commercially or to self-insure a group of exposure units was analyzed in an expected utility framework. It was shown how the expected gain or loss in utility from self-insurance could be obtained. Moreover, it was shown how to obtain computable upper and lower bounds on the expected gain or loss of utility from self-insurance on an *ex ante* basis. Since the owner of a business firm should insure if he (she) can increase his (her) expected utility of wealth, it was shown how to translate utility bounds into dollar bounds for the purpose of comparing costs. If the premium quoted is smaller than the calculated lower bound, the firm should unequivocably buy insurance; whereas, if the quoted premium is larger than the calculated upper bound, the firm should unequivocably self-insure. Between the two bounds, more information is needed before a rational decision can be made. Numerical illustrations for the decision rules were developed to compute the bounds for acceptable premiums.

Most importantly, it was shown that the upper and lower bounds used to calculate the range of acceptable premiums could be obtanied by probability distributions which did not depend upon the utility function utilized. This significant result means that the risk manager can make decisions without directly assessing the preference structure of the owner or owners of the firm at every possible wealth level. Risk managers probably could improve their decisions by adding these new scientific decision rules to their bag of tools. The model can also be used to obtain information concerning the number of exposure units that a risk-averse business firm should have for self-insurance to be financially feasible on a scientific basis.

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